A Primer on Optimal Mass Transportation and the Supremal Monge Problem

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Abstract

This seminar explores foundational concepts and central existence results in optimal transport theory. We transition from the intuitive but restrictive Monge problem to the Kantorovich relaxation, leveraging direct methods of calculus of variations to guarantee the existence of optimal plans. By constructing the dual problem and applying tools from convex analysis—specifically c-transforms and c-cyclical monotonicity—we establish strong duality. This primal-dual framework allows us to demonstrate the existence and uniqueness of optimal maps for strictly convex costs, culminating in Brenier's Theorem in $\mathbb{R}^d$. Finally, we resolve the supremal (L^∞) Monge optimal transport problem, rigorously proving that a secondary variational problem selects a unique optimal solution induced by a transport map.

The full notes for this presentation can be accessed from refrainfr.github.io

References

[1] L. Ambrosio, E. Brué, and D. Semola. *Lectures on Optimal Transport*. UNITEXT. Springer, Cham, 2nd edition, 2024.

[2] T. Champion, L. De Pascale, and P. Juutinen. The ∞-wasserstein distance: Local solutions and existence of optimal transport maps. *SIAM Journal on Mathematical Analysis*, 40(1):1–20, 2008.

[3] F. Santambrogio. *Optimal Transport for Applied Mathematicians*, volume 87 of *Progress in Nonlinear Differential Equations and Their Applications*. Birkhäuser/Springer, Cham, 2015.